Enunciados

Demuestra las siguientes identidades trigonométricas.

① $(1-\cos^2\alpha)\cdot\csc^2\alpha = 1$

② $\operatorname{sen}\alpha=\dfrac{(\cos\alpha)\cdot(\sec\alpha)}{\csc\alpha}$

③ $(\operatorname{tg}\alpha)\cdot(\operatorname{sen}\alpha)+\cos\alpha = \sec\alpha$

④ $\dfrac{\sec^2\alpha-1}{\sec^2\alpha}=\operatorname{sen}^2\alpha$

⑤ $(\sec^2\alpha)\cdot(\csc^2\alpha) = \sec^2\alpha+\csc^2\alpha$

⑥ $\operatorname{ctg}\alpha+\operatorname{tg}\alpha = (\sec\alpha)\cdot(\csc\alpha)$

⑦ $\dfrac{1}{1-\operatorname{sen}\alpha}+\dfrac{1}{1+\operatorname{sen}\alpha}=2\cdot\sec^2\alpha$

⑧ $\dfrac{1-\operatorname{sen}\alpha}{1+\operatorname{sen}\alpha}=(\sec\alpha-\operatorname{tg}\alpha)^2$

⑨ $\dfrac{\cos\alpha+\operatorname{sen}\alpha}{\cos\alpha-\operatorname{sen}\alpha}=\dfrac{\operatorname{ctg}\alpha+1}{\operatorname{ctg}\alpha-1}$

⑩ $\dfrac{\cos\alpha}{1-\operatorname{tg}\alpha}+\dfrac{\operatorname{sen}\alpha}{1-\operatorname{ctg}\alpha}=\operatorname{sen}\alpha+\cos\alpha$

Soluciones

Hay muchas maneras correctas de demostrar identidades trigonométricas. Aquí te mostramos una posibilidad para cada problema, que no tiene por qué coincidir con la que hayas escrito tú.

① $(1-\cos^2\alpha)\cdot\csc^2\alpha = \operatorname{sen}^2\alpha\cdot\dfrac{1}{\operatorname{sen}^2\alpha} = 1$

② $\dfrac{(\cos\alpha)\cdot(\sec\alpha)}{\csc\alpha} = \dfrac{\cos\alpha\cdot\dfrac{1}{\cos\alpha}}{\dfrac{1}{\operatorname{sen}\alpha}} = \dfrac{1}{\dfrac{1}{\operatorname{sen}\alpha}} = \operatorname{sen}\alpha$

③ $(\operatorname{tg}\alpha)\cdot(\operatorname{sen}\alpha)+\cos\alpha = \dfrac{\operatorname{sen}\alpha}{\cos\alpha}\cdot\operatorname{sen}\alpha+\cos\alpha = \dfrac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\cos\alpha} = \dfrac{1}{\cos\alpha} = \sec\alpha$

④ $\dfrac{\sec^2\alpha-1}{\sec^2\alpha} = 1-\dfrac{1}{\sec^2\alpha} = 1-\cos^2\alpha = \operatorname{sen}^2\alpha$

⑤ $\sec^2\alpha+\csc^2\alpha = \dfrac{1}{\cos^2\alpha}+\dfrac{1}{\operatorname{sen}^2\alpha} = \dfrac{\operatorname{sen}^2\alpha+\cos^2\alpha}{(\cos^2\alpha)\cdot(\operatorname{sen}^2\alpha)} =$

$= \dfrac{1}{(\cos^2\alpha)\cdot(\operatorname{sen}^2\alpha)} = \dfrac{1}{\cos^2\alpha}\cdot\dfrac{1}{\operatorname{sen}^2\alpha} = (\sec^2\alpha)\cdot(\csc^2\alpha)$

⑥ $\operatorname{ctg}\alpha+\operatorname{tg}\alpha = \dfrac{\cos\alpha}{\operatorname{sen}\alpha}+\dfrac{\operatorname{sen}\alpha}{\cos\alpha} = \dfrac{\cos^2\alpha+\operatorname{sen}^2\alpha}{(\operatorname{sen}\alpha)\cdot(\cos\alpha)} = \dfrac{1}{(\operatorname{sen}\alpha)\cdot(\cos\alpha)} =$

$= \dfrac{1}{\cos\alpha}\cdot\dfrac{1}{\operatorname{sen}\alpha} = (\sec\alpha)\cdot(\csc\alpha)$

⑦ $\dfrac{1}{1-\operatorname{sen}\alpha}+\dfrac{1}{1+\operatorname{sen}\alpha} = \dfrac{1+\operatorname{sen}\alpha+1-\operatorname{sen}\alpha}{(1-\operatorname{sen}\alpha)(1+\operatorname{sen}\alpha)} = \dfrac{2}{1-\operatorname{sen}^2\alpha} = \dfrac{2}{\cos^2\alpha} = 2\cdot\sec^2\alpha$

⑧ $(\sec\alpha-\operatorname{tg}\alpha)^2 = \left(\dfrac{1}{\cos\alpha}-\dfrac{\operatorname{sen}\alpha}{\cos\alpha}\right)^2 = \left(\dfrac{1-\operatorname{sen}\alpha}{\cos\alpha}\right)^2 = \dfrac{(1-\operatorname{sen}\alpha)^2}{\cos^2\alpha} =$

$= \dfrac{(1-\operatorname{sen}\alpha)^2}{1-\operatorname{sen}^2\alpha} = \dfrac{(1-\operatorname{sen}\alpha)^2}{(1+\operatorname{sen}\alpha)(1-\operatorname{sen}\alpha)} = \dfrac{1-\operatorname{sen}\alpha}{1+\operatorname{sen}\alpha}$

⑨ $\dfrac{\operatorname{ctg}\alpha+1}{\operatorname{ctg}\alpha-1} = \dfrac{\dfrac{\cos\alpha}{\operatorname{sen}\alpha}+1}{\dfrac{\cos\alpha}{\operatorname{sen}\alpha}-1} = \dfrac{\dfrac{\cos\alpha+\operatorname{sen}\alpha}{\operatorname{sen}\alpha}}{\dfrac{\cos\alpha-\operatorname{sen}\alpha}{\operatorname{sen}\alpha}} = \dfrac{\cos\alpha+\operatorname{sen}\alpha}{\cos\alpha-\operatorname{sen}\alpha}$

⑩ $\dfrac{\cos\alpha}{1-\operatorname{tg}\alpha}+\dfrac{\operatorname{sen}\alpha}{1-\operatorname{ctg}\alpha} = \dfrac{\cos\alpha}{1-\dfrac{\operatorname{sen}\alpha}{\cos\alpha}}+\dfrac{\operatorname{sen}\alpha}{1-\dfrac{\cos\alpha}{\operatorname{sen}\alpha}} = \dfrac{\cos\alpha}{\dfrac{\cos\alpha-\operatorname{sen}\alpha}{\cos\alpha}}+\dfrac{\operatorname{sen}\alpha}{\dfrac{\operatorname{sen}\alpha-\cos\alpha}{\operatorname{sen}\alpha}} =$

$= \dfrac{\cos^2\alpha}{\cos\alpha-\operatorname{sen}\alpha}+\dfrac{\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha-\cos\alpha} = \dfrac{\cos^2\alpha}{\cos\alpha-\operatorname{sen}\alpha}-\dfrac{\operatorname{sen}^2\alpha}{\cos\alpha-\operatorname{sen}\alpha} =$

$= \dfrac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\cos\alpha-\operatorname{sen}\alpha} = \dfrac{(\cos\alpha+\operatorname{sen}\alpha)(\cos\alpha+\operatorname{sen}\alpha)}{\cos\alpha-\operatorname{sen}\alpha} = \operatorname{sen}\alpha+\cos\alpha$